1А 23.10.12

$\Delta_n = p\Delta_{n-1} + q\Delta_{n-2}$

$t^2 - pt - q = 0$

t_1, t_2 са корените на уравнението

$t_1 + t_2 = p$

$t_1 t_2 = -q$ $\quad t_1 \neq t_2$

$\Delta_n = (t_1+t_2)\Delta_{n-1} - t_1 t_2 \Delta_{n-2}$

1) $\Delta_n - t_1\Delta_{n-1} = t_2(\Delta_{n-1} - t_1\Delta_{n-2})$

2) $\Delta_n - t_2\Delta_{n-1} = t_1(\Delta_{n-1} - t_2\Delta_{n-2})$

3) $\Delta_{n-1} - t_1\Delta_{n-2} = t_2(\Delta_{n-2} - t_1\Delta_{n-3})$

4) $\Delta_{n-1} - t_2\Delta_{n-2} = t_1(\Delta_{n-2} - t_1\Delta_{n-3})$

$\Delta_3 - t_1\Delta_2 = t_2(\Delta_2 - t_1\Delta_1)$

$\Delta_3 - t_2\Delta_2 = t_1(\Delta_2 - t_2\Delta_1)$

от 1) $\Delta_n - t_1\Delta_{n-1} = t_2^{n-2}(\Delta_2 - t_1\Delta_1)$

от 2) $\Delta_n - t_2\Delta_{n-1} = t_1^{n-2}(\Delta_2 - t_2\Delta_1)$ $\Rightarrow$ от 2) изваждаме 1)

$$\Delta_{n-1} = \frac{t_1^{n-2}(\Delta_2 - t_2\Delta_1) - t_2^{n-2}(\Delta_2 - t_1\Delta_1)}{t_1 - t_2}$$

$$\Delta_n = t_2^{n-2}(\Delta_2 - t_1\Delta_1) + \frac{t_1^{n-1}(\Delta_2 - t_2\Delta_1) - t_1 t_2^{n-2}(\Delta_2 - t_1\Delta_1)}{t_1 - t_2}$$

$$= \frac{t_1^{n-1}(\Delta_2 - t_2\Delta_1)}{t_1 - t_2} - \frac{t_2^{n-1}(\Delta_2 - t_1\Delta_1)}{t_1 - t_2}$$

$\Delta_n = t_1^n C_1 + t_2^n C_2$

$C_1 = \frac{\Delta_2 - t_2\Delta_1}{t_1(t_1 - t_2)}$ $\qquad C_2 = \frac{\Delta_2 - t_1\Delta_1}{t_2(t_1 - t_2)}$

при $t_1 = t_2 = t$ имеем

$$\Delta_n - t\Delta_{n-1} = t(\Delta_{n-1} - t\Delta_{n-2})$$
$$\Delta_{n-1} - t\Delta_{n-2} = t(\Delta_{n-2} - t\Delta_{n-3})$$
$$\vdots$$
$$\Delta_3 - t\Delta_2 = t(\Delta_2 - t\Delta_1)$$

$$\sum_{k=2}^{n-1} t^{n-k}(\Delta_k - t\Delta_{k-1}) = \Delta_n - t^{n-1}\Delta_2$$

$$\Delta_k - t\Delta_{k-1} = t(\Delta_{k-1} - t\Delta_{k-2}) = t^2(\Delta_{k-2} - t\Delta_{k-3}) =$$
$$= t^{k-2}(\Delta_2 - t\Delta_1) \Rightarrow \sum_{k=2}^{n-1} t^{n-2}(\Delta_2 - t\Delta_1) = \Delta_n - t^{n-1}\Delta_2$$

$$t^{n-2}(\Delta_2 - t\Delta_1) + t^{n-2}(\Delta_2 - t\Delta_1) - \dots - t^{n-2}(\Delta_2 - t\Delta_1) + t^{n-1}\Delta_2 = \Delta_n$$

$$(n-2)t^{n-2}\Delta_2 - (n-2)t^{n-1}\Delta_1 + t^{n-1}\Delta_2 = \Delta_n$$

$$\Delta_n = C_1 t^n + n C_2 t^n$$

$$C_1 = \frac{2t\Delta_1 - \Delta_2}{t^2} \qquad C_2 = \frac{\Delta_2 - t\Delta_1}{t^2}$$

Зад

1.4 а)

$$\Delta_n = \begin{vmatrix} 2 & 1 & 0 & 0 & \cdots & 0 \\ 1 & 2 & 1 & 0 & \cdots & 0 \\ 0 & 1 & 2 & 1 & \cdots & 0 \\ & & & & & \\ 0 & 0 & 0 & 0 & \cdots & 2 \end{vmatrix} \Rightarrow$$ разлагаме по 1-ви ред

$$(-1)^2 2\,\Delta_{n-1} + (-1)^3 1 \begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ & & & \\ 0 & 0 & \cdots & 2 \end{vmatrix} \Rightarrow$$ разлагаме по 1-ви стълб

$\Delta_n = 2\Delta_{n-1} - 1\Delta_{n-2}$ } местим Δ_n и заместване с t

$t^2 - 2t + 1 = 0$ след $D = ?$ и $t_{1,2} = ?$ $\Rightarrow$

$t_{1,2} = 1 \Rightarrow (t-1)^2 = 0$

$$\boxed{\Delta_n = t^n C_1 + n t^n C_2}$$

$\Delta_1 = |2| = 2$ $\quad \Delta_2 = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3$ { $(2 \cdot 2 - 1 \cdot 1)$ } кръстосано

Δ_2 се взема от 1-вите 2 реда и 2 колони на Δ_n

$$\left.\begin{aligned} \Delta_1 &= t^1 C_1 - 1 t^1 C_2 \\ \Delta_2 &= t^2 C_1 - 2 t^2 C_2 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} 2 &= C_1 + C_2 \\ 3 &= C_1 + 2C_2 \end{aligned}\right\} = \begin{aligned} C_1 &= 1 \\ C_2 &= 1 \end{aligned}$$

$$\Delta_n = 1 + n$$

1,7 б) $\Delta_n = \begin{vmatrix} 3 & 2 & 0 & \dots & 0 \\ 1 & 3 & 2 & \dots & 0 \\ 0 & 1 & 3 & 2 & \dots 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & 3 \end{vmatrix}$ развиване по 1-ви стълб

$\Delta_n = (-1)^2 3\Delta_{n-1} + (-1)^3 1$ $\begin{vmatrix} 2 & 0 & 0 & \dots & 0 \\ 1 & 3 & 2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 1 & 3 \end{vmatrix}$ развиване по 1-ви ред

$\Delta_n = 3\Delta_{n-1} - 2\Delta_{n-2}$ заместване с t =>

$t^2 - 3t + 2 = 0$

$D = 1$ $t_1 = 1$, $t_2 = 2$

$\Delta_n = t_1^n C_1 + C_2 t_2^n$

$\Delta_1 = 3$, $\Delta_2 = \begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix} = 3.3 - 2.1 = 7$

$\left.\begin{aligned} \Delta_1 &= C_1 1^1 + C_2 2^1 \\ \Delta_2 &= C_1 1^2 + C_2 2^2 \end{aligned}\right\}$ $\left.\begin{aligned} C_1 + 2C_2 &= 3 \\ C_1 + 4C_2 &= 7 \end{aligned}\right\} \Rightarrow \begin{aligned} C_2 &= 2 \\ C_1 &= -1 \end{aligned}$

$\Delta_n = 2^{n+1} - 1$

Детерминанта на Вандермонд

$W(x_1 x_2) = \begin{vmatrix} 1 & 1 & 1 & \dots & 1 \\ x_1 & x_2 & x_3 & \dots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \dots & x_n^2 \\ \dots & \dots & \dots & \dots & \dots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \dots & x_n^{n-1} \end{vmatrix}$ $\begin{matrix} \leftarrow -x_1 \\ \leftarrow -x_1 \\ \vdots \\ \leftarrow -x_1 \end{matrix}$

$$\begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ 0 & x_2-x_1 & x_3-x_1 & \cdots & x_n-x_1 \\ 0 & x_2(x_2-x_1) & x_3(x_3-x_1) & \cdots & x_n(x_n-x_1) \\ \vdots & & & & \\ 0 & x_2^{n-2}(x_2-x_1) & \cdots & & x_n^{n-2}(x_n-x_1) \end{vmatrix} \begin{matrix} \text{разв. по I-ви ред} \\ = (-1)^2 1 \end{matrix} \begin{vmatrix} x_2-x_1 & \cdots & x_n-x_1 \\ \vdots & & \\ x_2^{n-2}(x_2-x_1) & \cdots & x_n^{n-2}(x_n-x_1) \end{vmatrix}$$

$$\prod_{i=2}^{n}(x_i-x_1) \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ x_2 & x_3 & \cdots & & x_n \\ x_2^2 & x_3^2 & \cdots & & x_n^2 \\ \vdots & & & & \\ x_2^{n-2} & x_3^{n-2} & \cdots & & x_n^{n-2} \end{vmatrix} = \prod_{1\le i<j\le n}(x_j-x_i)$$

1,9 а)

$$\begin{vmatrix} 1 & 1 & \cdots & 1 \\ 1 & 2 & 3 & n+1 \\ \vdots & & & \\ 1^n & 2^n & 3^n & (n+1)^n \end{vmatrix} = (2-1)(3-1)\cdots(n+1-1)(3-2)(4-2)\cdots$$
$$\cdots((n+1)-2)\cdots((n+1)n) =$$
$$= n!(n-1)!(n-2)!\cdots 1!$$

$W(1,2,\dots,n+1)$ Вандермонд

1,9 б)

$$\begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ \sin\alpha_1 & \sin\alpha_2 & & & \sin\alpha_n \\ \sin^2\alpha_1 & \sin^2\alpha_2 & & & \sin^2\alpha_n \\ \vdots & & & & \\ \sin^{n-1}\alpha_1 & \sin^{n-1}\alpha_2 & & & \sin^{n-1}\alpha_n \end{vmatrix} = \prod 2\sin\frac{\alpha_j-\alpha_i}{2}\cos\frac{\alpha_j+\alpha_i}{2} =$$
$$1\le i<j\le n$$

$$= 2^{\frac{n(n-1)}{2}} \prod \sin\frac{\alpha_j-\alpha_i}{2}\cos\frac{\alpha_j+\alpha_i}{2}$$

$$1\le i<j\le n$$